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Temporal statistics of the light emitted by a bi-axial laser resonator

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Abstract

Power spectral density of the light emitted by a bi-axial CW Nd:YAG, diode-pumped, laser is investigated. We show that the beams diffracted by a slit with adjustable width, placed inside the cavity, against the folding mirror, can modify the temporal autocorrelation function and spectral behaviour of the laser emission. In a second experiment, carried out with a pulsed Nd:YAG laser, we show that the temporal profile itself depends on the diffraction by the slit placed inside the resonator. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

This paper deals with the analysis of diffraction effects on the spectral and temporal characteristics of light emitted by a bi-axial laser, when a slit with adjustable width is placed against the output coupler. Such diffraction effects have been used by several authors to synchronise the radiations emitted by two or more parallel amplifying media [1-4] in order to

increase laser emission power [4-6]. The experiments described in the following have been performed with resonators for which we studied the spectral or temporal periodicity in the laser emission.

In the first experiment, carried out with a CW Nd:YAG diode-pumped laser, we consider the statistical properties of the radiation by studying the autocorrelation function [7] of the laser field using a two beam interferometer. In the second one, we analyse, thanks to a streak-camera, the space and time behaviour of the field emitted by a flash pumped Nd:YAG laser. This is possible because of the strong increase in the instantaneous laser power due to pulsed emission.

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2. Temporal autocorrelation of a bi-axial CW laser emission with diffraction-induced mutual beam locking

Firstly, we describe the structure of the resonator, as well as the arrangement used to perform the autocorrelation of the laser field. Then, we study the influence of the width of the slit placed inside the resonator on the features of the modulus of the laser field autocorrelation function.

2.1. Structure of the resonator

The Fourier transform resonator with length L (Fig. 1) is composed of a lens L_1 (focal length L/2 = 0.5 m) and of two plane mirrors M_1 and M_2 located in the focal planes of the lens and perpendicular to the lens axis. In fact, the laser medium is a Nd:YAG crystal with one of its plane ends coated, thus constituting the end mirror M_1 of the cavity. The sources used for optical pumping are two CW laser diodes connected to optical fibers with a diameter equal to 150 μ m and a numerical aperture equal to 0.14. The beams emerging from the fibers illuminate the YAG crystal through the mirror M_1 . The

population inversion concerns two separate volumes inside the YAG crystal. The axes of these volumes are parallel and sufficiently distant (d = 2 mm) to avoid any coupling due to laser beam superposition. In each pumped volume the laser beam has a diameter and divergence corresponding to the optimal gain. The pump beams induce thermal distortions on laser waves, but we used moderated pump powers so that the distortions were cancelled by a translation of the lens L_1 . Thus, two TEM₀₀ beams are spontaneously generated without the need for another filtering aperture in the cavity. We placed against the mirror M₂ a slit F_2 whose width *a* is adjustable around 3 mm. Because the beam diameter is equal to 2 mm (at $1/e^2$), the slit does not affect seriously the Gaussian profile. The large sides of the slit are perpendicular to the plane containing the axes of the two pumping beams.

The laser radiation emitted through the mirror M_2 is therefore composed of two superimposed wavefronts, with distinct direction of propagation and corresponding to the two beams travelling in the opposite direction inside the laser cavity. For various widths *a* of the slit, the statistical properties of one of these beams are studied using the autocorrelation device described below.



Fig. 1. CW Nd:YAG bi-axial laser with a Fourier transform resonator used to test the influence of the diffracting slit F_2 (adjustable width *a*) on the spectral and temporal statistical properties of one of the two emitted beams.

2.2. Field autocorrelation

The autocorrelation device is shown in Fig. 2. The beam splitter BS and the mirrors M_3 ($R \approx 1$) and M_4 ($R \approx 1$) are the elements of a Michelson interferometer. The mirror M4 intercepts only half of the beam. The interferometer is set in such a way as to form fringes which are rectilinear and equidistant, for an interference order varying around the value $2L/\lambda_0$ where L is the optical length (about 1 m) of the laser cavity shown in Fig. 1 and λ_0 is the mean wavelength of the radiation emitted by this laser $(\lambda_0 = 1064 \text{ nm})$. A supplementary mirror M₅, parallel and identical to M_4 , is placed at the distance L behind M₄. Thus, we obtain a second field of interference fringes. The latter fringes are straight, parallel to the previous ones and present a periodicity which is almost identical, but the interference order varies around $4L/\lambda_0$. Both fringe fields obtained respectively with the M5 and M4 mirrors are juxtaposed on the matrix of a CCD camera and recorded simultaneously.

For each of the previous interferograms, the illumination E, measured along an axis perpendicular to the fringes is expressed versus the complex degree of coherence g(t') by [7]:

$$E = E_{a} \{ 1 + \text{Re}(g(t')) \}$$

where E_0 is a constant; t' is the delay introduced between the two interfering beams (around 2L/c or 4L/c; c being the speed of light in vacuum); and Re(g) is the real part of g.

The function g(t') is proportional to the autocorrelation function G(t') of the laser field U(t) defined by:

$$G(t') = \int_{-\infty}^{+\infty} U^*(t) U(t+t') dt,$$

where $U^*(t)$ is the complex conjugated of U(t).

The fringe visibility defined by $V = (E_{\text{max}} - E_{\text{min}})/(E_{\text{max}} + E_{\text{min}})$, is proportional to the modulus of the autocorrelation function G(t') previously defined. The fringes obtained with M₄ and M₃ give $|G_{\tau}(t')|$ whereas those obtained with M₅ and M₃



Fig. 2. Autocorrelation set-up used to analyse statistical properties of one of the two CW beams emitted by the laser.

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give access to $|G_{2\tau}(t')|$ with $\tau = 2L/c$. $G_{\tau}(t')$ is the autocorrelation function for the delay t' varying around τ .

By considering one of the two beams emitted by the laser we can measure, using this autocorrelator, the modulus period of the autocorrelation function of the optical field. By varying the width of the slit F_2 placed in the resonator, we would like to show that this period has either the value 2L/c, or the value 4L/c, for a given value of L.

2.3. Experimental results

Two waves fall on the mirror M_2 , their propagation directions making the angle $2\theta = 3.14 \times 10^{-3}$ rad. These waves can interfere on M_2 . The distance between two consecutive fringes is expressed by $p = \lambda_0/2\theta = 0.34$ mm. The central fringe is either bright (this transversal profile will be called 'limited cosine') or dark ('limited sine' profile), depending on the width *a* and on the position of the centre of the F₂ slit in its plane. For various widths *a*, we observed the illumination in the plane of F₂ as well as both interferograms given by the autocorrelator.

Fig. 3A is a recording of the 'cosine limited' profile, whereas Fig. 3α shows both corresponding fringe fields obtained by using the correlator, for the delays τ and 2τ . As the contrast of the fringe fields obtained for τ and 2τ is maximal, we can conclude that the modulus of the autocorrelation function includes a periodical component with the period τ .

If we vary the width of the slit F_2 by a quantity $\Delta a < \lambda_0/2\theta$, we note that the illumination in the exit plane of the laser equipped with the slit F_2



Fig. 3. Illumination in the plane of slit F_2 and corresponding fringe patterns given by the autocorrelation device for delays equal to $\tau = 2 L/c$ and 2τ ; L being the length between the laser mirrors.

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remains modulated by two beam fringes perfectly stable and highly contrasted. When we increase Δa , we note a sudden change in the fringe pattern existing in the plane of the slit F₂ towards a new fringe distribution described by a 'limited sine' profile with the same period p (Fig. 3B). These fringes remain stable and highly contrasted providing the supplementary variation Δa of the width does not exceed $\lambda_0/2\theta$. From the interferogram given by the correlator (Fig. 3β), we conclude that the modulus of the autocorrelation function of the optical field still shows a periodicity equal to τ .

If the width a of the slit F_2 is continuously varied, with the other parameters remaining constant, we observe a periodic change from a 'limited sine' profile to one that is 'limited cosine'.

It is not easy but possible to give the slit F_2 a value *a* which makes the probability of both profiles identical. Then, instead of the fringes, in the plane of the slit F_2 , we note the illumination characteristic of the fundamental profile TEM₀₀ (Fig. 3C). For this slit width, the correlator no longer gives any fringes for the delay τ , however we still observe highly contrasted fringes for the delay 2τ (Fig. 3γ). We conclude that the modulus of the autocorrelation function shows a period equal to 2τ instead of τ in the previous cases.

The earlier results have shown only that periods equal to τ or 2τ exist in the modulus of the autocorrelation function because the mirrors of the correlator were fixed. Then, we shifted the mirror M_4 along the correlator axis and we found highly contrasted fringes every 15 mm or so corresponding to a time period equal to 0.1 ns. A spectroscopic analysis of the laser radiation has confirmed the existence of a spectral modulation which is almost periodic, the period being approximately 10 GHz (Fig. 4). The number of these spectral laser lines depends on the strength of the pump power. This type of emission can be attributed to the effects of spatial hole-burning [8] enhanced by the position of the amplifying medium at the end of the resonator, against the mirror M₂. As these effects do not depend on the presence of the slit F_2 inside the resonator, we have not taken them into account in this paper. We have only made sure that the laser ran well above the threshold in order to obtain many lines emitted in the Nd:YAG bandwidth.



Fig. 4. Spectral modulation due to spatial hole burning.

2.4. Interpretation of results

Below, we propose a qualitative interpretation of the role played by the slit F_2 during the transient process leading to the synchronisation of the two beams which fall on to the coupler M_2 .

Let us consider, during this transient regime, two waves which are both emitted by each pumped volume P₁ and P₂. These waves result from spontaneous emission and their duration Δt is such that: $\Delta t \Delta f \approx 1$ where Δf is the frequency bandwidth of the laser medium. A₁ and A₂ are the complex amplitudes of the waves in P₁ and P₂ at time t = 0(Fig. 5a).

In order to simplify, we suppose that the centre of the slit F_2 is exactly located in the middle of the optical distance covered by the light starting from M_1 and coming back at M_1 . As the frequency bandwidth is narrow, the diffraction through the slit F_2 is identical to the diffraction of a monochromatic wave with a λ_0 wavelength.

After one roundtrip including the reflection on the mirror M_2 whose area has been limited by F_2 , the complex amplitudes A_1 and A_2 have respectively become A'_1 and A'_2 (Fig. 5b). A constant being omitted, A'_1 and A'_2 are given by:

$$A'_{1} = A_{2} + A_{1d}$$
$$A'_{2} = A_{1} + A_{2d}$$
$$A_{1d} = \frac{A_{1}\sin(\pi a\theta/\lambda_{o})}{\pi a\theta/\lambda_{o}}$$



Fig. 5. Coherent building of the transient laser field by diffraction through the slit F_2 : after one round trip (t = 2 L/c) in the resonator, the field complex amplitude A_1 (resp. A_2) has become A'_2 (resp. A'_1); A_{1d} (resp. A_{2d}) is the far field diffracted by F_2 receiving A_1 (resp. A_2).

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is the far field signal diffracted by the slit F_2 receiving the wave A_1 .

In the same way:

$$A_{2d} = \frac{A_2 \sin(\pi a \theta / \lambda_o)}{\pi a \theta / \lambda_o}$$

is the signal diffracted by F_2 receiving the wave A_2 .

The phase locking of the two waves falling on to the mirror M_1 can be attributed to an iterative process. Fig. 6 illustrates simply one of the stages of this process. By considering the various waves as being quasi monochromatic, we can draw (Fig. 6a) the phasors corresponding to A_1 , A_2 , A_{1d} , A_{2d} for a slit width such that the waves A_1 and A_{1d} on the one hand, and A_2 and A_{2d} on the other hand, are in phase. After a round trip in the resonator, the phase difference between A'_1 and A'_2 is lower than that of A_1 and A_2 . Consequently, after a few round trips between the mirrors M_1 and M_2 , the two laser fields falling on to M_1 have identical amplitudes and phases at the centre of the slit F_2 ; this is the 'limited cosine' profile. The laser optical field $U_+(t)$ emitted through F_2 along one or other of the two distinct directions is such that:

$$U_+(t+\tau) = U_+(t)$$
 where $\tau = \frac{2L}{c}$

Whereas the autocorrelation function of this field has the following property:

$$G_{\tau}(t') = G_{2\tau}(t')$$

This explains the existence of highly contrasted fringes observed for both delays τ and 2τ , whereas the illumination in the plane of the slit F_2 is of the 'limited cosine' type.

When the width of the slit F_2 causes a phase difference equal to π between A_{1d} and A_1 on the one hand, and between A_{2d} and A_2 on the other hand (Fig. 6b) the phase difference between A'_1 and A'_2 is higher than that between A_1 and A_2 . After a few round trips inside the resonator, the vibrations have the same amplitude and a phase difference equal to π at the centre of the slit F_2 . When the field is established the illumination in the plane of F_2 is of the 'limited sine' type. The new field U_- on the laser axis verifies the following relation:

$$U_{-}(t+\tau) = -U_{-}(t)$$

In the same way, for the autocorrelation function of U_{-} , we obtain:

 $G_{\tau}(t') = -G_{2\tau}(t')$

As $|G(\tau)| = |G(2\tau)|$, both fringe patterns given by the correlator for the delays τ and 2τ still have identical visibilities, but with illuminations proportional to 1 + Re (g(t')) for t' approximately equal to



Fig. 6. Phasor diagrams showing the transient laser fields building: (a) A_1 (resp. A_2) and A_{1d} (resp. A_{2d}) are in phase; the resultant phasors show that the phase difference between A'_1 and A'_2 is lower than between A_1 and A_2 . (b) Phase difference between A_1 (resp. A_2) and A_{1d} (resp. A_{2d}) is equal to π ; the resultant phasors show that the phase difference between A'_1 and A'_2 is greater than between A_1 and A_2 .

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 2τ and 1 - Re(g(t')) for t' approximately equal to τ .

From this result we can conclude that the autocorrelation functions of the optical field, obtained for the 'limited sine and cosine' profiles do not have the same periodicity and that it exists a fringe shift between the two interferograms given by the autocorrelator for the delays τ and 2τ . It is not easy to observe this shift (Fig. 3α and 3β) because the fringe periodicities are not exactly the same.

Let us choose the width a of the slit F_2 in order to cancel the signals A_{1d} and A_{2d} diffracted towards the axes of the pumped volumes. This is possible by giving the width *a* exactly the same value as $m\lambda_0/\theta$, where m is an integer. The mirror M_2 only reflects both incident beams without introduction of synchronising diffracted signals. If we neglect the spontaneous emission, we can therefore attribute to each of the beams emerging from M_2 , a periodic temporal profile whose period is equal to 2τ . We recall that τ is the duration of a round trip in the resonator. For the delay τ , the fringe visibility is proportional to the cross correlation $G(t') = \int_{-\infty}^{+\infty} U^*(t) U(t+t') dt$: this integral is equal to zero because the correlation time of laser light, equal to the inverse of the laser emission bandwith is much shorter than the periodicity 2τ of the laser field. This explains firstly the absence of fixed fringes in the plane of the slit F_2 as shown in Fig. 3C, and secondly the cancellation of the autocorrelation function for the delay τ (Fig. 3 γ).

It must be noted that this fringe disappearance is due to the addition of illuminations on the CCD matrix during the time necessary for the recording of an image. In fact, there are transient fringes which could theoretically be observed in the frame of a spatial and temporal analysis providing the temporal resolution is higher than the inverse of the frequency bandwidth involved in the laser process. For the CW regime and low power radiation, this analysis cannot be done using devices existing today because the number of photons per degree of freedom is not sufficient. That is why we carried out an experiment in the pulsed, free running regime, with a laser resonator similar to the one shown in Fig. 1 and with a flash lamp as pumping light. With such a laser the number of photons per degree of freedom is sufficient to be recorded by a streak camera.

3. Coherence properties of a Nd:YAG pulsed laser beam

3.1. Resonator

The new resonator is shown in Fig. 7. A flash lamp is used as pumping source instead of laser



Fig. 7. Flash pumped laser used to analyse, in the nanosecond range, temporal behaviour of laser emission for various widths a of the slit.

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Fig. 8. Streak camera recording of laser emission (sine or cosine profile case) in an image plane of the M_1 mirror.

diodes in Fig. 1. As the Nd:YAG volume is entirely illuminated by the flash light, it is necessary to perform a modal filtering in order to select the same transverse profile (TEM $_{00}$) as in the CW regime. For this purpose we place two spatial filters inside the resonator. One of the filters, called F_1 in Fig. 7, is an opaque screen with two apertures selecting two distinct beams in the amplifying medium. The diameter of each aperture is equal to 0.2 mm and the distance between the centre of the apertures is $2\theta f = 1.42$ mm (f being the focal length of lens L_1). The other filter is the slit F₂, with varying width, placed against the mirror M₂. Because of the light diffracted by the slit the two waves selected by the filter F_1 can be synchronised. We varied the width of the slit F_2 in a restricted range so that the laser waves filtered by F₁

and F_2 have a transversal profile of the type TEM₀₀ as in the case of the CW laser (Fig. 1).

3.2. Experimental results

While the laser was emitting wave packets, in the free running regime, we formed an image of the two apertures of the filter F_1 on the entrance slit of a streak camera, in order to obtain the temporal profile of the corresponding beams.

When the field, inside the slit F_2 , was modulated by stable and highly contrasted fringes, we observed each time two synchronous beams emitted through M_2 . The power emitted in the above conditions had the features of a periodic noise, the period being equal to the duration 2L/c of one round trip in the



Fig. 9. Streak camera recording of laser emission for precise widths a of the slit giving no fringes across this slit.

resonator (Fig. 8). This noise may be considered as the superposition of the different longitudinal modes, oscillating with random phases.

However, for very accurate widths of the slit F_2 equal to $m\lambda_o/\theta$, with *m* integer, we observe firstly the cancellation of the fringes inside the slit F_2 and secondly the emission of signals shown in Fig. 9. The two beams emitted through the mirror M_1 have identical intensities. But they are no longer synchronous: the temporal shift is equal to $\tau = 2L/c$. Each beam has a period of $2\tau = 10.6$ ns equal to twice the duration of one round trip in the resonator. This observation validates the model proposed in the CW regime: without synchronising signals produced by the slit F_2 , the light undergoes a simple change in direction because of reflection by M_2 . The laser then behaves like an in-line laser of the Fabry–Perot type, with an optical length equal to 2 *L*.

4. Conclusion

We have shown that power spectral density of the light emitted by a folded CW laser pumped by two laser diodes can be modified by placing, in the cavity, a diffracting slit with varying width. We attribute this behaviour to the synchronisation (or not) of the two beams by the slit on the beam folding mirror.

With a pulsed laser the emitted power is sufficient to be measured by a streak camera. Thus, the temporal period of the emitted signal is most often equal to the duration τ of one round trip in the cavity. But for particular values of the slit width, which has to be very accurately adjusted, the observed period becomes equal to 2τ . In the latter case, the illumination distribution in the plane of the slit shows no fringe at all: this is a classical scheme where statistical intensity fluctuations are superposed in energy rather than in amplitude, during the recording time of the detector like in the CW emission. However, if spatial and temporal energy distributions existing in the plane of the slit were analysed in real-time, for instance thanks to a streak camera, temporal sequences of fringe patterns should be observed, each of them having an average life-time equal at least to the inverse of the frequency bandwidth. Such an observation gives access to instantaneous interferences, with their randomly distributed contrasts and phases. In spite of its stochastic features, this process could be called a 'perfectly coherent' one, as exhibiting space-time dependent interferences, the structure of which may be completely understood and described in the coherent Fourier optics frame.

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